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In memoriam Paul Kunitzsch

REVIEWS

Arabic Section

Mathematical geography in the western Islamic world: geographical coordinates of localities in the al-Maghreb and al-Andalus localities (9th-18th centuries)

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Abstract: The analysis of Maghrebi and Andalusi geographical coordinates extracted from the book by Kennedy & Kennedy's «Geographical coordinates of localities from Islamic sources» (1987) allows a renewed vision of mathematical geography in these regions. We show the major role of al-Khwārizmī (9th century) in correcting Ptolemy's erroneous data and in creating the Arab system. In particular, we show that it was this scholar who introduced what would later be called the «meridian of water». The subsequent authors (37 sources examined) made global and local improvements that were very generally unsuccessful over time.

Key-words: geographical coordinates, meridian of water, Ptolemy, al-Khwārizmī, longitude of the Mediterranean.

Introduction

Arabo-Islamic civilization inherited from Ptolemy (c.90-c.168) a complete mathematical geographical system which was based on the following principles:

(1) the inhabited lands occupy half of the terrestrial sphere, so their longitudes are spread over 180°;
(2) the «zero» meridian (= prime meridian) is located at the westernmost part of the Fortunate Islands (Canary Islands); and
(3) the latitudes are counted from the equator.

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Arab geographers, following al-Khwārizmī (c. 780-c. 850), realized that Eurasia occupied less space on the globe than Ptolemy thought. They therefore had to establish new coordinates. Measuring the latitude of a locality is relatively easy: an observation at night with an astrolabe, or a quadrant, or at noon on a sundial, allows an excellent evaluation (cf. Evans, 1998, p. 33). On the other hand, measuring longitude is a major scientific problem. The evolution of evaluation methods can be summarized as follows (see for example: Evans, 1998; Comes, 1995, 2000; Jullien 2002):

(1) The oldest method is by dead reckoning: knowing the size of the globe and the orientation and length of the straight line joining two localities, we can calculate the differences in coordinates and in particular the longitude component.[1] This is obviously a very imprecise method because of the difficulty of accurately estimating distances and azimuths ashore.

(2) The simultaneous observation and timing of cosmic events, including lunar eclipses, from two locations on Earth also makes it possible to evaluate the time difference and therefore the longitude component of the distance between the two locations. But these are rare circumstances and, in addition, they must be planned in advance in order to schedule astronomical observations. This method is mentioned in particular by al-Battānī (858-929) (Delambre 1819 p. 16) and described, among others, by al-Bīrūnī (Kennedy 1973 p.100) and al-Marrākushī (13th century) (in Sédillot, 1834, p. 312-314) but it was already known at the time of Hipparchus (2nd century B.C.) (see Comes 2000 for a complete review).

(3) Galileo (1564-1642), developed a method similar to the previous one but based on the observation of Jupiter's satellites. The obstacle of the rarity of the event then disappears but the method requires the use of an astronomical telescope.

(4) It was the invention in England in the 18th century of reliable mechanical clocks over several weeks that really solved the problem in a practical way. Indeed, these marine watches made easily accessible knowledge of the time difference between two localities, without requiring astronomical observation.

1. As formalized by al-Bīrūnī (10th -11th century): see Kennedy (1973).

Ptolemy, like the Muslim geographers of the period of interest to us (9th to 18th century), had only the first two methods. It is therefore be expected that longitudes will be somewhat imprecise. The progress made by Arab scholars, compared to the geography of Ptolemy,[2] will therefore be slow, uncertain and inchoate.

In Ptolemy's system, the 90° meridian, which is supposed to correspond to the middle of the world, has become very important.[3] It is known as the «Dome of Arîn» (Qubbat Arīn) or «Dome of the Earth». The *de facto* existence of two fundamental meridians, spaced 90° apart, has proved problematic when it comes to modifying the longitudes of Ptolemy. It is indeed possible to keep fixed one or the other of fundamental meridians, indiscriminately.[4] In fact, it is certain that the positions of the «zero» meridian and the «Dome of Arîn» have changed several times over the course of history. We know, for example, that the «zero» meridian has moved away from the Canary Islands to become the «Water Meridian»[5] (Comes 1994, 2000) and that the available sources do not agree on the position of the «Dome of Arîn». The details of these benchmark changes remain extremely obscure (see Kennedy, 1996-p.188; Tibbetts, 1992; Sezgin, 2000-2007 and Sezgin, 2010-v3, p.1-33; for attempts at synthesis; see also King 2014).[6] Moreover, another element adds to the confusion: it is the fact that Arabic geographers have used

2. Ptolemy's error is due to an underestimation of the Earth diameter, and, probably, to an overestimation of the terrestrial distances localized in distant lands in relation to its base: Alexandria. See on this subject: Tupikova & Geus (2013), and Russo (2013).

3. See Pellat (1986).

4. This is, in particular, what al-Bīrūnī (Kennedy 1973, p. 91) and al-Marrākushī, (13th century) (in Sédillot, 1834, p. 312-3) assert, but in reality, they use a third solution; it is to choose any fixed meridian, in the Middle-East, and move the two reference meridians. It is this solution that was, it seems, initiated by al-Khwārizmī and adopted by all the authors (see discussion below).

5. This name did not appear explicitly until 1321 in a Western book («Portuguese Almanac of Madrid»), but it is indeed the same concept that it is about (Comes 1994).

6. Historically, it is also interesting to consult the discussions on the position of the «Dome of Arîn» which opposed Biot (1841) and Sédillot (1842) following the publication of the works of Sédillot (1834) and Reinaud & de Slane (1840). Moreover, it seems that the Arab authors themselves were a little lost in these multiple and mobile reference meridians. Thus, for example, the data from al-Marrākushī are compatible with a «Dome of Arîn» located at 80° from the «zero» meridian (in Sédillot 1834, p. 318), and al-Bīrūnī (11th century) retains 100° (Kennedy and Kennedy 1987, p. 196). However, their respective «zero» meridians differed by only 10° and above all, by definition, and in any case, the longitudinal distance had to be 90°.

two subsystems (named A and C by Kennedy & Regier 1985) which are offset by 10° from each other, imposing the choice of different «zero» meridians at the level of the Canary Islands (type C) or Atlantic Cost (type A). The origin and reasons for the coexistence of these two ways of expressing longitude remain very uncertain (Kennedy & Regier 1985, Tibbetts 1992).

In this paper, we propose to re-examine the whole problem of the historical evolution of longitudes, but limiting ourselves to the Western Mediterranean region, and particularly to the Maghreb, al-Andalus[7] and very close areas. The sources we will use are all included in Kennedy & Kennedy's (1987) huge compilation of ancient geographical coordinates, which are hereafter identified as K&K.

The compilation by Kennedy & Kennedy (1987) (= K&K).

In 1987, Edward S. and Mary H. Kennedy published a book entitled *Geographical coordinates of localites from Islamic sources*. It is a 723-pages book that compiles more than 12,000 coordinates for localities all over the old world from the works of Ptolemy and from 72 different Islamic sources. These sources correspond either to manuscripts or to modern works reproducing medieval geographical tables. Each source contains the coordinates of a variable number of localities (2 to 656). Some of these localities are cited and located by only one source, but others by several dozen (Jerusalem: 65 mentions, Medina and Mecca 63; Baghdad 60...).[8]

K&K proposed a chronological ranking of all sources. As most of them are dated, a solid chronological framework is available to study the evolution of longitudes from Ptolemy to the 18th century. Parallel to this chronology, K&K and then Regier (1987) statistically established the degree of similarity between the sources. These two approaches make it possible to characterize the influence of certain sources on more recent ones.

7. There are many references to localities on the northern shore of the Mediterranean, and even of the whole of Europe, in K&K; but they seemed to us, a priori, less reliable. We preferred to limit our analysis to places that are easily accessible to Muslim travelers and scholars.

8. The most cited cities in the Muslim West are Kairouan with 29 mentions, Tripoli 28, Cordoba 27, Tunis 24, Tangier and Fez 23, Madhia 22...

Curiously, this mass of digital data has been little used by historians of geography. At most, we can quote Robles Macías (2014) has used a small number of Arabic sources in his study on the «longitude of the Mediterranean» (KHU, BIR, ZAY, TOL, TAJ and MAG; see below for the meaning of these codes).

The data and methods used

Given the limited geographical scope of this study, we first extracted from the K&K pages, all available data correspond to the following current countries: Spain, Portugal, Morocco, Algeria, Tunisia, Libya and very close regions such as the Mediterranean islands. We have added to this list, for comparison, a few localities in the Middle East that are mentioned many more than 35 times. The only sources used are those for which K&K clearly indicates whether they are type A or type C. After having deleted incomplete coordinate pairs, we have a computer database of 1646 «cards» concerning 191 localities. The sources that appear in this selection are listed in Table 1. This list refers to 39 of the 74 sources listed by K&K; the others contain only oriental localities or do not have a type (A or C) clearly determined.

Table 1: Chronological list of sources used (39 of the 74 sources in the K&K compilation); Computer Code (4 or 7 signs) and dating. The list order and dates are taken from K&K, which will be consulted for further details.

An anonymous Ẓāhirīyah treatise	ZAH		C
The Tuḥfa-i Sulaymānī an anonymous work	THF	c. 1668	C
An astrolabe in the British Museum	AST	c. 1710	C
Ain-i-Ākbari of Abū l-Fazl-i-ʿĀllami	AIN	c. 1580	C
Ulugh Beg's Zīj-i Sulṭānī	ULG	c. 1440	C
Al-Kāshī's Zīj-i Khāqānī	KAS	c. 1420	C
An anonymous table in Gotha MS 1467	GT2	before 1467	C
Ibn al-Shāṭir's al-Zīj al-jadīd	ʿSHA	c. 1350	A
The Zīj of Shams al-Munajjim	MUN	c. 1330	C
The Ashrafī Zīj of Sayf-i Munajjim	ASH	1310	A

An anonymous Zīj rreported by Abū l-Fida’	ZDJ-FID	before 1310	A
The qiyâs reported by Abū l-Fida’	QYS-FID		A
Ibn al-Bannā’’s Minhāj al-ṭālib	BAN	before 1321	A
The Zīj of Jamāl ad-Dīn al-Baghdādī	BAG	c. 1285	A
al-Marrākushī’s Jāmiʿ al-mabādi’	MAR	c. 1250	C
A Zīj by Muḥyī al-Dīn al-Maghribī	MAG	1276	C
A table in Leiden Ms. Cod. 1001(15)Warn.	LYD		A
Naṣīr al-Dīn al-Ṭūsī Zīj-i Īlkhānī	TUS	c. 1270	C
Tāj al-azyāj by Muḥyī al-Dīn al-Maghribī	TAJ	1258	C
Al-Tūqānī’s Risāla fī l-ʿamal bi l-asṭurlāb	TUQ		C
Ibn Saʿīd as reported by Abū l-Fidā’	SAA-FID	before 1286	A
Ibn Saʿīd’s Kitāb basṭ al-arḍ	SAA	before 1286	A
A Zīj reported in Yāqūt’s Muʿjam	ZIJ-YAQ	before 1220	A
The anonymous Marseilles tables	MRS	12th cent.	A
The list in the Toledan Tables	TOL	c. 1185	A
The Sanjarī Zīj, Istanbul Copy	SNH	c. 1120	A
The Sanjarī Zīj, British Museum Copy	SNB	c. 1120	A
al-Zayyāt’s Kitāb al-aqālīm	ZAY	before 1058	A
Bīrūnī’s Qānūn reported by Abūl-Fidā’	BIR-FID	c.1040	A
al-Bīrūnī’s Al-Qānūn al-Masʿūdi	BIR	c. 1040	A
Al-Dimyāṭī’s Maʿrifat al-qibla	QBL		C
Ibn Yūnus’s Al-Zīj al-Kabīr al-Ḥākimī	YUN	c. 990	A
al-Battānī’s Al-Zīj al-Ṣabi’	BAT	c. 900	C
The anonymous Kitāb al-Aṭwāl	ATH-FID		A
ʿAjā’ib al-aqālīm al-sabʿah by Suhrāb	SUH	c. 930	A
Kitāb Rasm al-Maʿmūr, from Abūl-Fidā’	RES-FID	c. 820	A
The list attributed to al-Khuwārizmī	KHU	c. 820	A
The Geography of Ptolemy	PTO	c. 140	C
The Handy Tables of Ptolemy	HTP	c. 140	C

In order to homogenize the data and be able to compare them, we have taken into account the specificity of the longitudes provided by the sources classified A and we have systematically added 10°. In the end, the «cards», which can be extracted and classified according to any criterion, contain the following information:

> Name of the locality, Longitude (homogenized), Latitude, Source code,[9] date (possibly approximate).[10]

The geographical area studied is clearly under-represented in geographical tables. K&K have identified more than 2500 locations, and more than 12,000 pairs of coordinates. We study here only 7.5% of the localities, 14% of the geographical data which are provided by only 52% of the sources. It will therefore only be a first approach, the conclusions of which will have to be confirmed by a broader study.

In this study, we preferred to have a historical approach rather than develop global statistical methods that proved, finally, to be irrelevant (see Regier 1987). We have developed specific computer tools to graphically represent the data. The most useful of these automatically builds the evolution curve of each coordinate over time for any location in the database (= spectrum). As an example, Figure 1 shows the longitude spectrum for Tangier (Morocco). We now stress that this spectrum should not be «over-interpreted», indeed, it is certain that some of the fluctuations observed are simply due to occasional transcription errors in successive copies of manuscripts. The existence of this type of error can be demonstrated by the latitude spectrum (Fig. 2). This shows aberrations that cannot be explained by another hypothesis since, as we have already pointed out, a measurement error is unlikely on this parameter. The problem is that recopying errors can affect both longitudes and latitudes randomly and independently, so there is no way to determine whether or not a longitude data is affected by this type of error. We will therefore remember that for a trend, or a one-time fluctuation, to be considered significant, it must necessarily be observed in many localities.

9. As it appears in K&K, i.e. in the form of a 3 or 7 characters code.

10. That is, the datation indicated by K&K or, when this information is not provided, an approximate age evaluated according to the relative datation proposed by these authors. Examination of Table 1 shows that in this case the margin of approximation never exceeds a handful of decades.

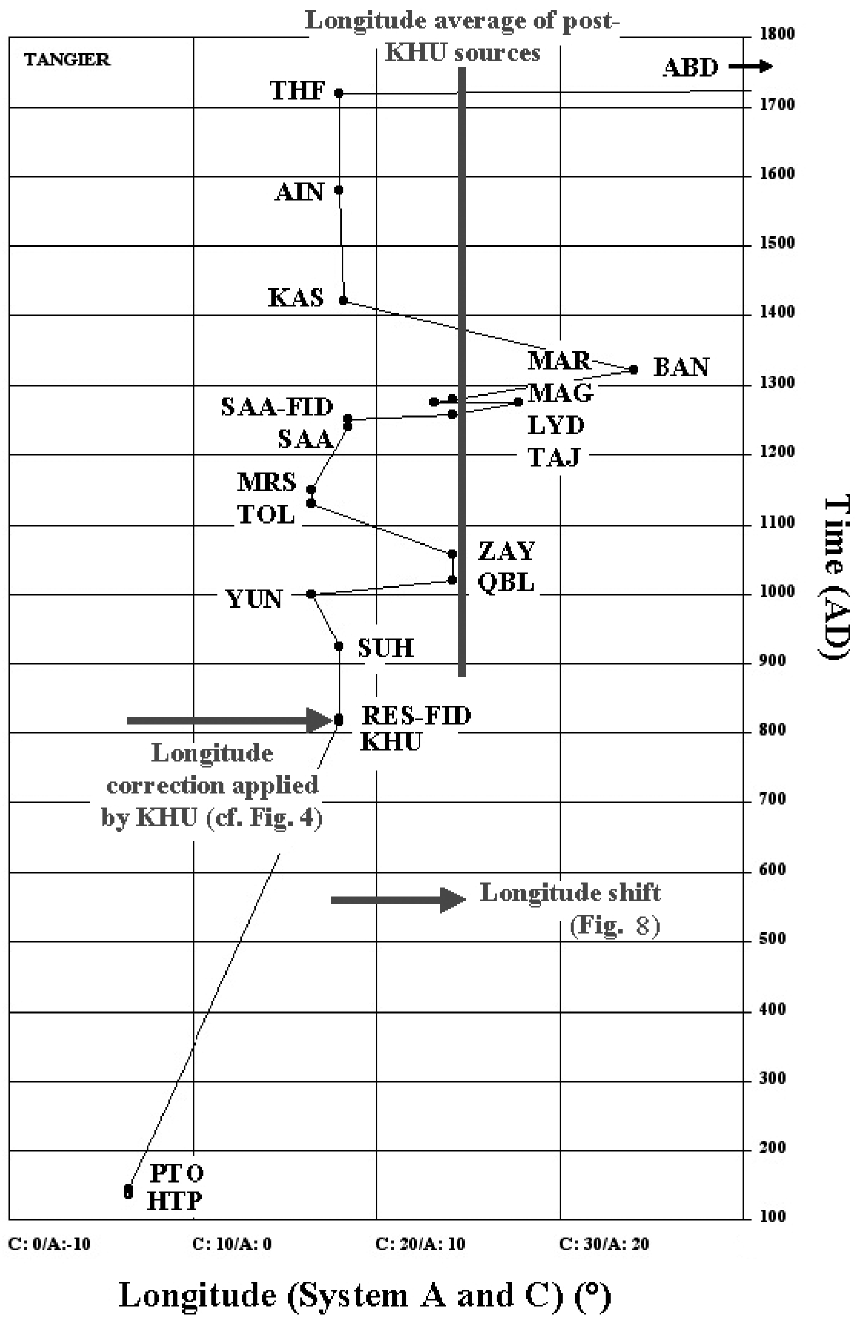

Figure 1: Spectrum of longitudes for the city of Tangier. The dots indicate the 21 references to this city found in the sources listed in Table 1. The lines joining the dots have a purely chronological

meaning. Few fluctuations observed are probably due to alterations in the original handwritten tables during the copying and transmission processes (scribal errors). The main parameters used in this study are indicated and illustrated in this example.

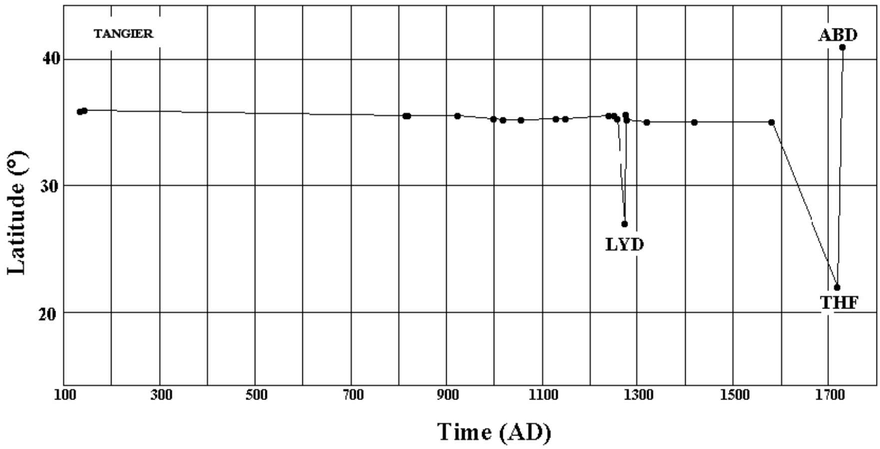

FIGURE 2: Spectrum of latitudes for the city of Tangier.
The fluctuations observed are very probably due to scribal errors.

RESULTS AND DISCUSSIONS

The example in Figure 1, which is very representative of the spectra obtained, makes it possible to schematize the historical evolution of longitudes. First, there is a significant correction in the value of Ptolemy by al-Khwārizmī, and then there is a series of sources that seem to fluctuate around the value of al-Khwārizmī. We will see that among these sources, few deserve special consideration; these are: YUN, QBL, ZAY and BAN.

The errors of Ptolemy (c.90-c. 168).

It is well established that the longitudes of Ptolemy are tainted with error if compared to current data (Gosselin 1790; general presentation in Dilke 1987, Tupikova & Geus 2013). As already mentioned, these mistakes are in part due to an underestimation of the diameter of the Earth, so the error over any longitudinal distance is generally proportional to it. (Fig. 3).

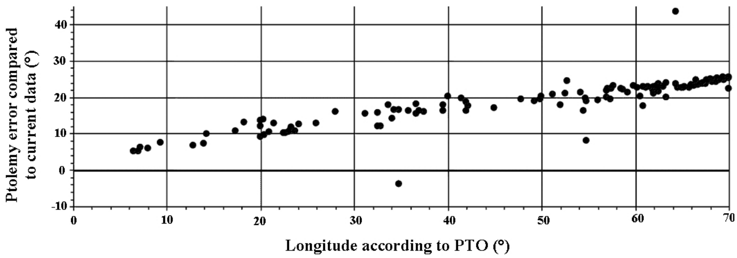

Figure 3: Ptolemy's error on the longitudinal distance between the different localities and his prime meridian (vertical axis) as a function of the localities' longitudes (according to PTO : horizontal axis); from the whole K&K data set.

The corrections of al-Khwārizmī (c.780-c.850)

Al-Khwārizmī is a very important scientist (Brentjes, in Hockey 2007, p. 632) who lived in Baghdad and was linked to the «House of Wisdom» of Caliph al-Ma'mūn (reign: 813-833). He was active in mathematics, astronomy and geography. His work in the latter field is known by a unique manuscript kept at the University of Strasbourg (MS 4-247), which was published by von Mzik (1926). The manuscript reference in the library catalogue can be translated as follows: *Translation of the Ptolemy Geography Treatise by Abū Ja'far Mohammad ibn Mūsā al-Khwārizmī*. In fact, it is more than a simple translation insofar as the geographical coordinates (longitudes and latitudes) of Ptolemy have been significantly modified by al-Khwārizmī (Fig.4).

In particular, and taking into account the 10° correction mentioned above and applied to the longitudes of this author, there is a general increase in longitudes of about 12° to the West of present-day Tunisia, and to the East, a more moderate increase that eventually cancels out in the Middle East. It is certain that al-Khwārizmī has not re-evaluated the longitudinal distance of the Canary Islands from the most western localities of Morocco and al-Andalus; the increase in longitude of these localities therefore corresponds to the displacement of the «zero» meridian by a few degrees to the West. Here we are witnessing the introduction of the «water meridian»,[11] even if the term will only appear later.

11. This concept was introduced in 1994 by Mercè Comes (1949-2010) from the University of Barcelona, the credit goes to her. In detail, some of the hypotheses proposed by this author will be questioned later in this paper. Moreover, it should be noted that al-Khwārizmī (first author to have used subsystem A according to K&K) has never used the Atlantic coast as a prime meridian. This

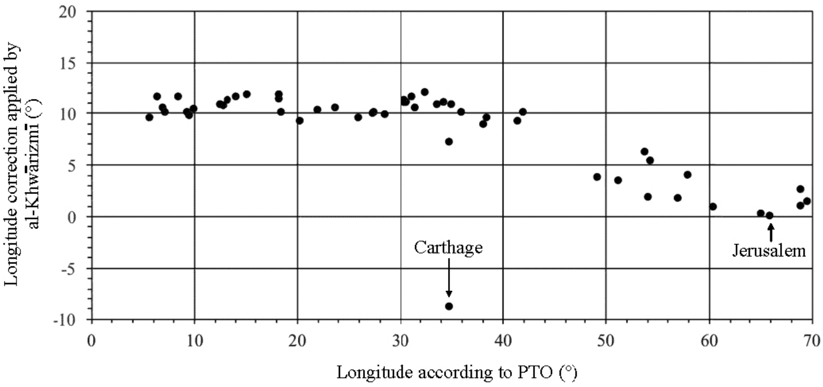

FIGURE 4: Corrections made by al-Khwārizmī (KHU) to the longitudes of Ptolemy (PTO) (vertical axis) as a function of the longitudinal position of the localities (according to PTO: horizontal axis). The point corresponding to Carthage, is clearly aberrant and due to a mistake of KHU.

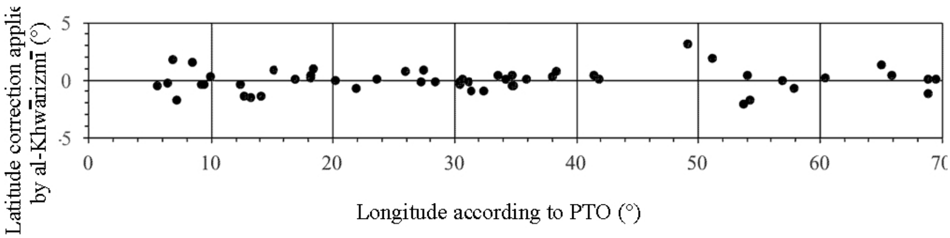

FIGURE 5: Corrections made by al-Khwārizmī (KHU) to the latitudes of Ptolemy (PTO) (vertical axis) as a function of the longitudinal position of the localities (according to PTO: horizontal axis).

But al-Khwārizmī's corrections do not stop there, indeed if this had been the case, figure 4 would have much less scattered points on either side of the average curve. Clearly, al-Khwārizmī completed his revision of Ptolemy's data with subtle corrections of a few degrees, more or less, that affect longitudes, as well as latitudes (Fig. 5). Hypotheses can be made about what justifies these two types of corrections:

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observation has important consequences on the origin of subsystems A and C (Mercier, *in preparation*).

(1) First of all, at the time of Caliph al-Ma'mūn, scientists of the «House of Wisdom» carried out several times, evaluations of the length of a terrestrial degree. They found values very close to those provided by Eratosthenes (3rd century B.C.) (Kennedy 1996) and above all larger than that retained by Ptolemy. Al-Khwārizmī therefore knew that the terrestrial globe was larger than Ptolemy thought, which is what justifies the narrowing of the longitude range between the Atlantic and the Middle East.

(2) moreover, at the time of al-Khwārizmī, the Muslim armies' conquest of North Africa and al-Andalus had been over for several decades, so it can be assumed that first-hand information on the latitudes and land distances of cities in this part of the world was available.

What is particularly remarkable in these corrections is that al-Khwārizmī avoids the trap of choosing a pre-existing reference meridian. Indeed, rather than counting its new longitudinal distances from the meridian of the Canary Islands or the «Dome of Arîn», he chose not to disturb the longitudes of the Middle East and *de facto* retained a fixed meridian[12] in this region, simply shifted by a «round» number of degrees (10°). Apparently, this is the Jerusalem meridian, but a full analysis of data from the Middle East would be required to confirm this attribution.

Nevertheless, it should be noted that some of Ptolemy's most important errors are not corrected. Thus, Ptolemy had considered that the Mediterranean coast of North Africa was globally rectilinear; he had not identified the inflexion that exists between Tunisia and the Gulf of Sirte.[13] Al-Khwārizmī made the same mistake by considering, for example, that Tabarka (coastal city, on the current border between Algeria and Tunisia), Carthage (suburb of Tunis) and Tripoli (in Libya) have exactly the same latitude (32°).

In the end, as far as longitudes are concerned, the corrections of al-Khwārizmī are quite successful, to a few degrees, in a large part of the area studied. There are only

12. The term «fixed meridian», in the sense that we use this term here, does not apply to a «zero» meridian; but to a meridian that keeps the same longitude as that assigned by the previous author(s) (here Ptolemy) and from which the new longitudes are calculated.

13. On the possible causes of this Ptolemy mistake see : De Rouire (1884), Vycichl W. (1969), Laporte (2003), Saada (2016). In addition, it should be noted that the World Map of al-'Umari (14th century) shows a correct layout of the Gulf of Sirte. It is therefore difficult to follow the late Professor Fuat Sezgin who presented this map as directly inspired (copied) from a Caliph al-Ma'mūn and Al-Khwārizmī's map.

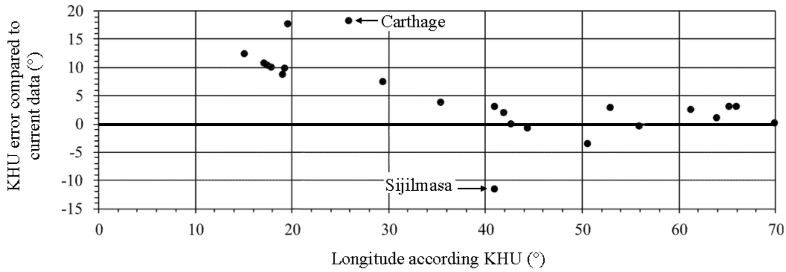

FIGURE 6: Errors in the longitudinal distance between Jerusalem and different localities in the study area (vertical axis), as a function of the longitudinal position of these localities (according to KHU: horizontal axis). The localities represented are those mentioned by KHU and whose modern equivalent has been identified by K&K. These localities, from west to east, are: Ghadira, Seville, Italiqa, Tangier, Asturiqa, Cordova, Granada, Carthage, Haykal, Sitif, Sijilmasa, Kairawan, Tunis, Thabraqa, Hamma, Tripoli, Barqa, Patras, Alexandria, Asqalan, Jerusalem, Damascus. The points corresponding to Carthage and Sijilmasa, are clearly aberrant and are due to a punctual poor positioning of these two localities by al-Khwārizmī.

significant errors left to the west of its 25° meridian (Fig. 6). This residual error will be corrected by only some of its successors; this is what we will consider now.

The successors of al-Khwārizmī.

The longitude spectra of figures 1 and 7 illustrate with some examples that sources after al-Khwārizmī do not agree with al-Khwārizmī and between them. For some localities (e. g. Tangier, Cordova, Murcia, Barqa...) the coexistence of two different traditions, characterised by values of longitude that differ from 5 to 11°, can be recognised throughout the period considered. Thus, for example, in the case of Tangier, 11 sources give us a longitude that deviates only slightly from 18°, and 6 others located around 25°. But other spectrum patterns are possible; thus, Kairawan's spectrum pattern is very clustered around an average value close to 41° while Tripoli's shows a fluctuation of nearly 10° without distinguishing well identified traditions. I will not propose explanations for the existence of these different patterns in this work, it would be worth further investigation.

It can be seen in Figure 1 that no post-KHU source adopts a longitude value in the order of Ptolemy's. This observation is absolutely general: detailed examination of the various sources used in this study, shows that no author derogates from it, and in particular, nobody returns to the old system with a «zero» meridian at the level of the Canary Islands. This means that al-Khwārizmī's successors have retained the fundamental principle of the latter; namely: to make corrections of longitude by reference to a fixed meridian located in the Middle East.[14] It is therefore clear that all sources after al-Khwārizmī refer to the «water meridian».[15] Comes (1994) inventoried many additional written sources that support this conclusion, but apparently did not realize that al-Khwārizmī had already imposed the displacement of the «zero» meridian at sea.[16] It should also be noted that in detail, the precise location of this meridian has never been unanimously accepted, as shown by the fluctuation in the longitude of most localities, including the most western ones. (Fig. 1 and 7). We will discuss this issue again.

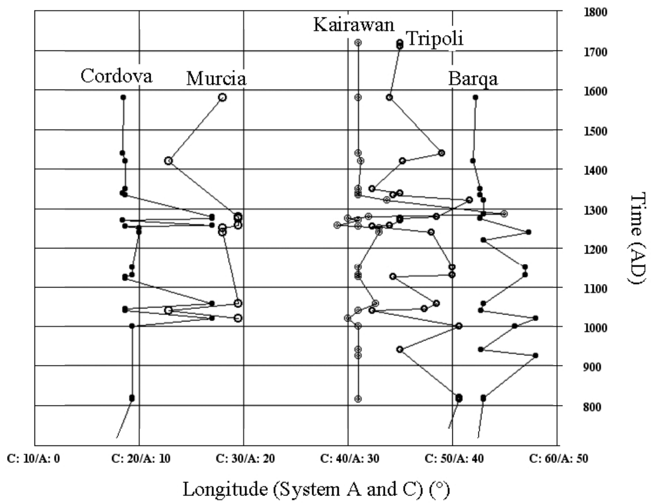

Figure 7: Some examples of spectra selected from the study area.

14. It is not certain, however, that all the authors chose, as al-Khwārizmī, to «sanctuarize» the meridian of Jerusalem, other solutions could be adopted from time to time. This uncertainty will not be resolved in this paper, it would require a specific study focusing on the Middle East.

15. Without necessarily being aware of it, or at least explicitly mentioning it: see below about al-Zayyāt.

16. Comes (1994, 1995, 2000) considers that it was al-Zayyāt, the first geographer to use the meridian of water.

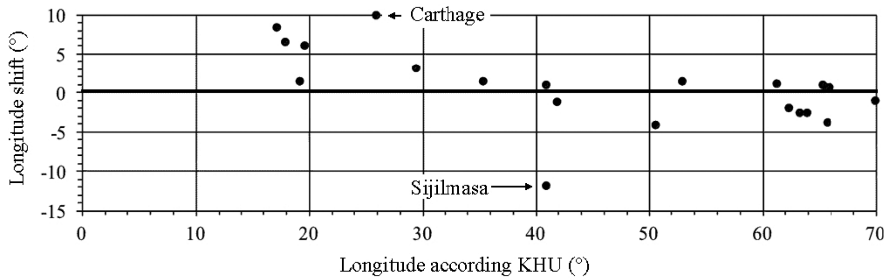

FIGURE 8: Shift between the longitude of KHU (al-Khwārizmī) and longitude average of post-KHU sources (vertical axis) as a function of the longitudinal position of the localities (according to KHU: horizontal axis). The localities represented are those mentioned both by KHU and by at least 5 post-KHU sources. These localities, from west to east, are: (number of post-KHU sources used in the calculation of the average): Seville (11), Tangier (17), Cordova (22), Granada (11), Carthage (5), Haykal (5), Sitif (5), Kairawan (25), Sijilmasa (22), Tunis (16), Tripoli (23), Barqa (17), Alexandria (46), Asqalan (36), Ramla (35), Jerusalem (55), Damascus (51).

Comparing figures 6 and 8, it can be seen that al-Khwārizmī's successors corrected his error on Sijilmasa, but not enough on Carthage (see text). It is also noted that the corrections concerning the most western localities are insufficient.

If we consider, for each locality, only the average of the longitudes provided by sources after KHU. The difference, for each locality, between these averages and the value given by KHU corresponds to the average correction (Fig. 8). It can be seen that this correction is generally negligible in the Middle East, however, it appears a break towards the 25° meridian. So, a dichotomy appears between the Western and Eastern Mediterranean with, in the East, longitude values close to those of al-Khwārizmī. In connection with this kind of observation, Comes (1995, 1997, 2000) assumed that, at least for some sources, the longitudes of the East and West Mediterranean were measured from different reference meridians. This idea is not supported by any period sources, and is in total opposition to the very foundations of mathematical geography, so we will not retain it. In fact, a detailed analysis shows that, once the KHU correction has been applied, the longitudinal distances between the Maghreb localities and Middle East city are relatively correct, except for those located in far West where the correction remains insufficient. So, the dichotomy between the East and West of the Mediterranean, with regard to longitude corrections by Muslim geographers, is easily explained by the fact that the dichotomy already existed in the data to be corrected (those of al-Khwārizmī).

We have seen that the authors were influenced by al-Khwārizmī in the choice of their fixed meridian, this influence is also sensitive by the fact that some errors, introduced by this author, are found in his successors and have never been corrected. For example, al-Khwārizmī curiously located Carthage more than 15° East of Tunis while these ruins are 0.1° West. The few more recent sources who mentioned Carthage, left it West of Tunis at longitudinal distances varying from 3 to 8°. Other errors may have been corrected at some point, but this correction is not systematically repeated by subsequent sources. We will take as an example an error of Ptolemy and al-Khwārizmī already mentioned, that of the latitude of the northern coasts of Tunisia and Libya (Fig.9). We can see that the correction was made quickly (source QBL, early 11th century, if this date is correct, see below) but that then the error will reappear regularly. Finally, some of KHU's errors were corrected by almost all the sources, such as Sijilmasa, which al-Khwārizmī had located in what would become Tunisia, while it was located south of Fez (Morocco). This mixture of influence between sources,[17] and random consideration of progress, seems to us to be an important feature of this entire period.

We have seen that al-Khwārizmī's corrections were insufficient in the far West of the Maghreb and that, clearly, some of his successors had realized this and had applied new corrections which are illustrated on the left of figure 8. If we now look at these sources independently of each other, we can identify the sources that contribute most to these additional corrections. The oldest ones are YUN, QBL and ZAY.

The tables of Ibn Yūnus (YUN- late 10th century), Zayn al-Dīn al-Dimyāṭī (QBL- 10-11th century?) and al-Zayyāt (ZAY- middle 11th century).

Ibn Yūnus, al-Dimyāṭī, and al-Zayyāt are the oldest of a series of sources that propose, for localities close to the Atlantic, longitudes much higher than those of al-Khwārizmī. No link or influence between these three sources can be established; they are independent (K&K).

17. On the dependence of sources on each other, see K&K's statistical analysis, and Regier (1987).

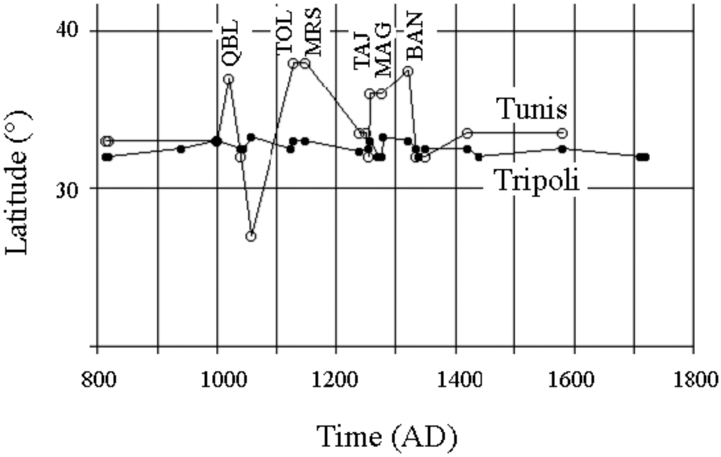

FIGURE 9: Latitude spectra of Tunis and Tripoli. The latter city is more southern by 4° than Tunis, which al-Khwārizmī and many subsequent sources had ignored. The few latitude values that can be considered as correct for Tunis, are indicated by their sources.

Ibn Yūnus is a famous astronomer who lived in Egypt (King 2008); his method of determining longitude was studied by Rufus (1931), it is the second method presented in the introduction (measurement of local sidereal time during a Moon eclipse).

Very little is known about Zayn al-Dīn al-Dimyāṭī, the only known manuscript of his *Kitāb* (QBL) is not dated but K&K places it chronologically between YUM and ZAY.[18]

Al-Zayyāt (c.951?-c. 1062) is an Andalusian geographer whose main work *Ḏikr al-Aqālīm,* was published and studied by Castelló (1989). In this text, al-Zayyāt does not explain the longitude determination technique he used. This is all the more surprising as the theory of the determination of the other geographical coordinates is discussed at length in a chapter on the relations between latitude and the declination of the sun according to the time of year (Zodiac) (Castelló 1989, pp. 44-47).

18. According to King (1984, p. 115, footnote 70) Zayn al-Dīn al-Dimyāṭī is a 12th century Egyptian author. For her part, Comes (1994 p. 48) considers that he is a 17th century author. In this paper, I have chosen not to deviate from K&K's views, but the question of QBL dating is open.

These three sources provide broadly the same longitudes as al-Khwārizmī for the Middle East, but much higher values for the western edge of the Mediterranean; this amounts to decreasing the longitudinal length of the Mediterranean, or «Longitude of the Mediterranean».[19] Robles Macías (2014), used a standardized method, using a large number of localities, to characterize the «Longitude of the Mediterranean» of the different Islamic and European sources he studied (his figure 2). We have preferred to develop a calculation simpler method that is better adapted geographically to the data we use here. The figure 10 illustrates the alternative method used: the «Longitude of the Mediterranean» is defined here by the longitudinal distance between the average of the longitudes of 3 cities chosen in the West and 4 cities in the East. These localities are frequently mentioned in the sources studied. This approach makes it possible to define two meridians characteristic of the eastern and western extremities of the Mediterranean. The horizontal lines in Figure 11 indicate the longitude of the two extreme meridians and the «Longitude of the Mediterranean», for each source studied. Not surprisingly, all sources use almost identical longitudes in Eastern, and a great variability of western longitudes.

This figure is completed by Table 2 which gives for each source:

(1) the value in degrees of the «Longitude of the Mediterranean»,
(2) the error (in %) compared to the current data.[20] It is noted that YUN, QBL and ZAY provide a value very close to the current value. Other later sources are in the same case (TAJ, LYD, MAG, and MAR),[21] most are statistically close to ZAY and probably result from it (K&K).
(3) the value of the westward movement of the «zero» meridian in relation to its location in PTO. In agreement with Comes (1994, 1995, 2000), the latter

19. In the words of Robles Macías (2014), to which this part of the study is an extension :«... longitude of the Mediterranean (...) will always mean the degrees (...) that separate the Iberian Peninsula and Morocco from the Asian continent, and not the distance in kilometers or miles between such regions».

20. Despite the use of a different definition, these results are in line with Robles Macías's values. To compare the values obtained here (PTO 46 ± 2%; KHU 23 ± 3%, BIR 22 ± 3%; ZAY 5 ± 1%; TOL 21 ± 5%... and so on) with those of Robles Macías (PTO 45 ± 4%; KHU 25 ± 3%, BIR 15 ± 7%; ZAY 7 ± 10%; TOL 22 ± 4%... and so on).

21. ZAH does not seem to be part of this list as it is a very late table, probably influenced by modern Western geography (K&K).

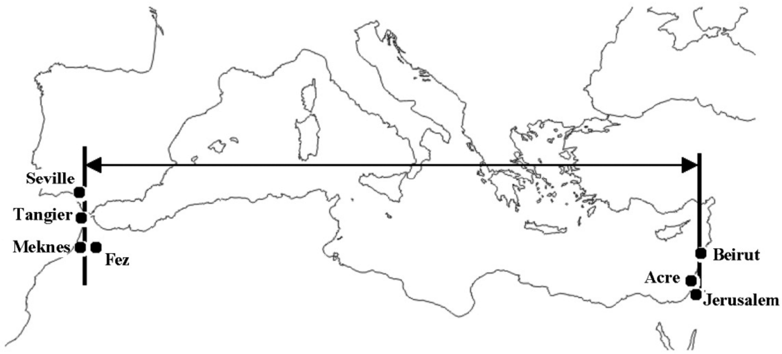

FIGURE 10: Definition of the extremities of the Mediterranean adopted in this study.

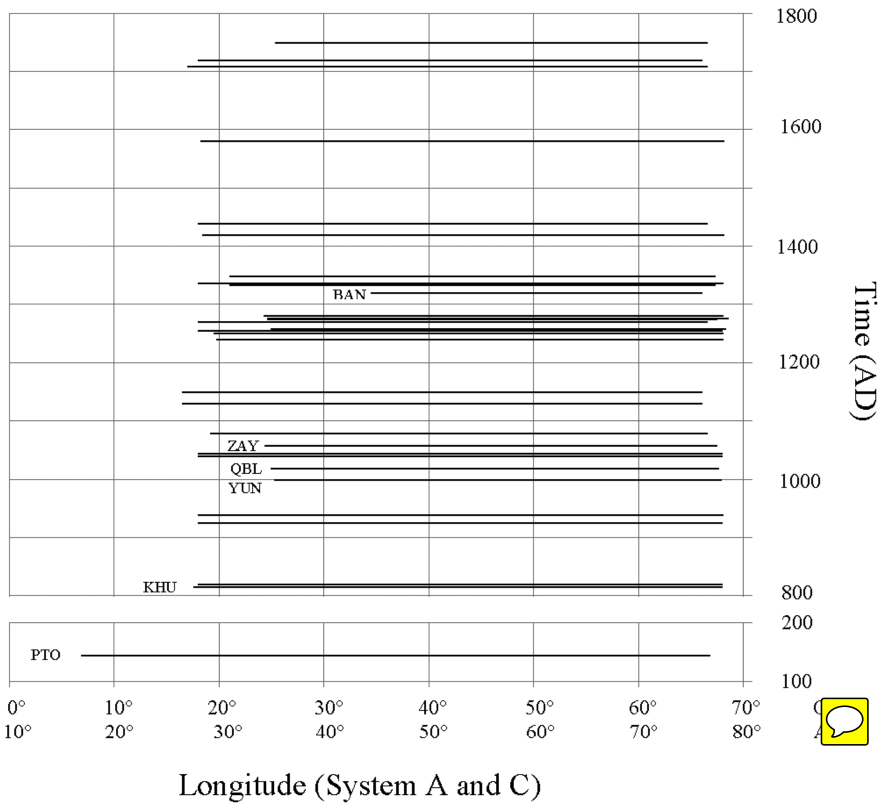

FIGURE 11: Position of the Mediterranean on the longitude grid, and longitudinal length, according to the sources studied. The chronological order is the same as on Table 2.

value corresponds, for a given source, to the difference in longitude between that source and PTO, for the most western localities. We can see that this value is close to 17.5° (= 17° 30') for about seven sources (including YUN, QBL and ZAY).[22] Comes (1994, 2000) gives a lot of importance to this value, probably because it has passed through the Alphonsine Tables in the Medieval Europe. But Table 2 shows that, no more than the correct value of the «Longitude of the Mediterranean», this value has been unanimously accepted; sources that are compatible with the value of 17.5° are rare: these are still TAJ, LYD, MAG, and MAR. In fact, a significant majority of later sources adopt a lower value (between 9.5° and 14°) and one is based on a significantly higher value; it is BAN (Ibn al-Bannā') with 23.2°, and which merits some comments.

Table 2 : Complements to Figure 10 (see text).

.	Longitudinal length (°) of Mediterranean sea	Error in reference to the modern value (%)	Displacement of PTO «zero/prime» meridian (°)
Modern	41,0	0.0	-
ZAH	41.2	0.3	18.5
THF	48,0	17.0	11.1
AST	49.5	20.7	10.1
AIN	49.8	21.5	11.4
ULG	48.5	18.2	11.1
KAS	49.7	21.1	11.6
SHA	46.3	12.7	14.1
MUN	50,0	21.9	11.1
ASH	46.3	12.7	14.1
BAN	31.5	-23.2	27.6
MAR	43.8	6.6	17.4
MAG	43.9	7.0	17.7
LYD	42.8	4.4	17.7
TUS	48.5	18.2	11.1

22. Curiously, al-Zayyāt does not seem to realize this implication because in his text he continues to define the «zero» meridian as Ptolemy did (Castelló 1989, pp. 56 & 88).

TAJ	43.3	5.6	18.1
TUQ	49.9	21.7	11.1
SAA-FID	48.5	18.2	12.6
SAA	48.2	17.6	12.9
MRS	49.5	20.7	9.6
TOL	49.5	20.7	9.6
ZAY	43.0	4.9	17.5
BIR-FID	49.9	21.7	11.1
BIR	49.9	21.7	11.1
QBL	42.7	4,0	18.1
YUN	42.6	3.8	18.4
ATH-FID	50.0	21.9	11.1
SUH	49.9	21.7	11.1
RES-FID	50,0	21.8	11.1
KHU	50.3	22.7	10.8
PTO	59.9	46.0	0.0

The table of Ibn al-Bannāʾ (1256-1321)

Ibn al-Bannāʾ al-Marrākushī al-Azdī is best known for his work in mathematics. He nevertheless wrote some astronomical works which were studied by Vernet (1952). In chapter 4 of his *Minhāj al-ṭālib li-taʿdīl al-kawākib* (Vernet 1952 p. 75) he provides us with a table of coordinates for 32 localities (BAN in K&K), but he does not comment on it and gives us no indication of how it was established. K&K and Regier (1987) indicate that this table is, statistically speaking, largely independent of the others.

Ibn al-Bannāʾ's geographical table (BAN) is characterized by high longitude values in the West. As the BAN values for the Middle East are very close to the other sources, it is concluded (1) that Ibn al-Bannāʾ is in line with his predecessors with regard to the meridian of water and respect for the longitudes of the Middle East, and (2) that he presents us with a Mediterranean whose longitudinal length is significantly reduced compared to the other sources and current data (Fig.9 & 10).

We can only regret that he did not leave us information on the methods he used to achieve this result, but at least we have an indication of his motivations. Indeed, regarding the calculation of the *qibla*, Samsó (2007) notes that «... surprisingly enough, this astronomer rejected the use not only of the imprecise methods of folk astronomy but also of those of spherical astronomy». One of the reasons given by Ibn al-Bannā' is that «the differences in geographical longitude between Mecca and other Islamic cities were not reliably known». With such an opinion on previous works in mathematical geography, we can understand that he has been led to propose a heterodox solution...

Conclusions

Developed civilizations need geographical coordinates, if only for navigation and trade. In the case of the Islamic civilization, this need is also motivated by the calculation of the *qibla* which depends, when a scientific method is chosen, on the geographical coordinates of the place and those of Mecca. Muslim scholars inherited Ptolemy's system and then improved it for their use. In Ptolemy's system, the «zero» meridian was located in the Canary Islands. Al-Khwārizmī realized that Africa, among other things, did not occupy as much space on the globe as Ptolemy thought it did. It was therefore necessary to correct the longitudes of different localities. Remarkably, he chose to preserve the ancient longitudes in the Middle East (with a correction of 10°) and, as a result, to move the «zero» meridian (which later became what would be called the «water meridian») to the Ocean. The following authors have applied (A) or not (C), the 10° correction but none will deviate from this principle.

Progress in the calculation of longitude and hence the reduction of the longitudinal distance between the western localities (Maghreb and al-Andalus) and those of the Middle East, started with al-Khwārizmī (9th century).

His corrections amend quite well Prolemy's mistakes between the Middle East and present-day East Algeria. Further West, al-Khwārizmī has applied the same correction as in the current Tunisia (11°-12°). That was not enough! The additional corrections were made independently by Ibn Yūnus (late 10th century), Al-Zayyāt (middle 11th century), and perhaps Zayn al-Dīn al-Dimyāṭī if the dating of this source is correct,[23] who found an almost exact size for the Mediterranean.

23. See footnote 18

It is clear that al-Khwārizmī inspired almost all subsequent sources which sometimes added improvements. These improvements can be local or global. With regard to the local plan, these improvements have often only been temporary; they have been repeated by the following authors only occasionally. On the global level, several 9th and 10th century sources have therefore proposed, independently of each other, a longitude of the Mediterranean very close to the current data, but, again, their proposals were rarely included in subsequent sources. It is therefore not really possible to consider that the 14th-17th century sources are significantly better than al-Khwārizmī.

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